11-2014

Feasibility of Scalable Quantum Computers

Benjamin N. Goodberry
Southeastern University - Lakeland

Feasibility of Scalable Quantum Computers

Benjamin N. Goodberry

Dr. Gordon Miller

HONR 4112

1 November 2014

Table of Contents

Feasibility of Scalable Quantum Computers

Introduction

The advent of quantum computing has brought about a change in how scientists think about information systems. If we can create a computer that has limitless potential to expand and answer complex problems, what is between current technology and that goal? And are there fundamental limits to how powerful a quantum computer could become? While working small quantum computers can be found now, the results they obtain are so small that they could be done not only by a classical computer, but often also by hand. It is obvious that if quantum information grows in importance, it will have to become large enough to answer problems efficiently that cannot be answered right now.

Problems faced in quantum computers come about as a result of fundamental properties of physics. Precise measurement of a qubit destroys any entanglement that qubit has with another. Acting even on qubits which haven't been measured introduces error into their states. These are not problems faced by classical computers. The 1 or 0 that a classical bit represents is simple and it is obvious that a value close to 1 would represent 1 in physical computers. That is not so with quantum computers- if two values are obtained from running a computation twice, it is possible for neither, one, or both of them to be valid solutions.

Scaling a quantum computer comes with even greater burdens. The threshold theorem puts forth stringent requirements for a computer to be scalable and as a computer gets bigger, so do errors between qubits, and errors from the environment. If these errors are too great, the computer does not satisfy the threshold theorem, and therefore cannot be made scalable. Here is where there is question of whether a quantum computer of any useful size can be created: is there any way to minimize errors enough that a sufficiently large computer can provide accurate results? Also, will minimizing that error remove any benefits of the computer's quantum rather than classical nature?

Creating scaled quantum computers could introduce huge possibilities in math, physics, engineering, and computer science. Such issues as the many-body problem would be efficiently solved, a result known to be impossible with classical computers. Current encryption protocol would become useless, as Shor's algorithm can be used by a large computer to factor large numbers within relatively short amounts of time. Simulation of the most fundamental quantum systems would be possible, and all without prohibitively large computers. Not only could quantum computers reasonably answer any of those questions, it could do so without taking exponentially longer times or taking significantly more computation power.

Currently, a large gap exists between the theoretical quantum computers we could create and the experimental ones that have shown success. While trivially small numbers can be factored, and one-dimensional potential systems can be simulated, those systems are redundant with modern classical computers. Attempts to create a physical, scaled quantum computer have so far all failed. Even the largest successes have been moot in the overall view of computer science. The bridge between where quantum computation is now and where it needs to be to become viable in commerce, personal use, or government use must be bridged by attempting to address errors and build bigger systems.

Many possible solutions exist to the creation of a scalable quantum computer. So far, experimental computers have been severely limited by number of qubits available, environmental error, or resource costs to build the computers themselves. It is entirely possible that using some of the more recent ideas, such as using the structure of diamonds, might be the key to realizing the goal of scalable quantum computers.

Chapter 1

Overview of Quantum Computation Schemes

Though many quantum computing schemes have been attempted, the most common few represent the vast majority of research done in quantum computation. The primary types will be discussed here in terms of their feasibility while scaling to larger systems as well as efficiency in monetary cost and space required. Most research done focuses specifically on how the computer itself will behave when extended. However, in many cases it fails to address whether such results are truly significant. While some scheme might be technically feasible, if it is prohibitively expensive in terms of its physical resources, it is not worth considering for scaling in commercial or consumer uses as it will never become advanced enough to see whether it scales smoothly. For this reason, scalability both in terms of economic and technical possibility will be addressed.

Three systems stand out among all quantum computation schemes, in two categories. The first category includes optical quantum computers, which have significant support and research surrounding possible physical implementation. However, fewer experimental results have come as a result of studying photonic qubits. Optical systems have several aspects indicating increased scalability over other methods, so these will be addressed. The second category includes two systems: using individual electrons as qubits, with either single or double quantum dots. The former uses one electron to represent a quantum state, while the latter has two entangled electrons placed next to each other and analyzes their combined spin using electric charge. All three systems have been proven in principle experimentally, and these will be the primary methods used to discuss feasibility of scaling quantum computers in general.

Optical Schemes

Optical quantum computers show significant promise in comparison to most other computation schemes. Photons are used as the computer's qubits, which is a desirable trait, since controlling singular qubits is simpler in photonic computers than many other systems, as no prohibitively expensive equipment is required to create photons with the properties desired. Generally, the qubit must be created with specific pure-state polarization, be kept in a specific location, and have minimal interaction with other qubits or the environment before entanglement. If any of these aspects is not met, it will not hold accurate enough information to provide accurate results.

The orthonormal states these photons create are found in their polarization; since any polarized photon is a linear combination of arbitrarily-defined horizontal and vertical polarity, it can be represented in any location on the Bloch sphere. These photons, then, are sufficient for output of quantum states as computational results. The other required aspect, that the qubits have the ability to be entangled and manipulated in their entangled state, is inherently present as proved by Lu *et al.* where entanglement was proven to the extent of 12 standard deviations. The strong entanglement demonstrated by optical qubits indicates the feasibility of the method in general, as the first step of Shor's algorithm is essentially trivial so long as the qubits involved are highly entangled.

The fact that Lu *et al*. devise a system using an optical system that demonstrated sufficient entanglement proves the computation system is quantum rather than classical. Every step of Shor's algorithm is carried out successfully in their experiment, and the end results have very high (>99%) accuracy. Scaling is also implied, since the system utilizes cluster states, which can be expanded endlessly. O'Brien comes to a similar conclusion, that linear optical systems or cluster states are the most likely to retain entanglement and least likely to have error that is prohibitively large for a scalable computer.

O'Brien discusses the implications of optical quantum computing at length, focusing specifically on long-term scaling and the barriers faced in their realization. He states that one of the primary issues with feasible photonic qubit schemes is the need for several independent sources of electrons for entanglement to occur. One solution he proposes, which has been tested to a small degree elsewhere, is the use of a diamond. Such a system would be a cross between linear optical systems and semiconductors, reducing the issues of both systems. As O'Brien describes large systems, he states, "The majority of experimental demonstrations to date have relied on non-scalable single photon sources…scaling to useful devices will require high efficiency single photon sources and detectors that are efficiently coupled," indicating the primary issues facing scaling are restriction to single photon emitters and insufficient entanglement.

Uskov *et al.* address the need for additional resources to increase the likelihood of successful outputs in a linear-optical quantum computer. Their results indicate that adding ancillary qubits to an optical scheme does nothing to increase maximum probability of success.

So, unlike semiconductors, resources do not grow exponentially simply because of the requirement for ancillary qubits. While this is a moderate advantage in the scaling properties associated with photonic qubits, since quantum computers are generally approached on how to decrease exponential size increases, this has limited impact. Ancillary qubits are, at worst, only polynomially less efficient in use than standard entangled qubits. Even though the information indicates fewer resources are required for such a system, it has only moderate impact once the system is scaled.

Many issues with scalability are addressed more effectively by optical systems than any other. First, no ancillary qubits are theoretically required to scale a quantum computer, greatly reducing the cost required (as use of ancillary qubits tends to be inefficient). Second, as previously mentioned, cluster states may be used effectively by optical computation schemes. The benefits of these cluster states are apparent in previously mentioned research by O'Brien: many qubits can be acted upon simultaneously without substantial increase in computation time or coherent error. Also, given that the qubits will be used for the same step of the computation process, they can be recycled in appropriate error correction schemes. Third, photonic qubits are readily created by inexpensive technology, since photons are ubiquitous, unlike such things as semiconductors which tend to be expensive and hard to access in large quantities. All three of these properties contribute to a lower entry barrier for scaling a quantum computer using photonic optical qubits.

Semiconducting Schemes

Perhaps the most widely studied method of experimental quantum computers is the semiconductor. Among these, the most common versions utilize silicon chips and gallium arsenide. Since a scheme based on semiconductors requires an electron donor, Calderón, Koiller, and Das Sarma offer sulfur, selenium and tellurium as viable options. The authors prove that using any of these provides a very long time frame before error is insurmountable, so semiconductors seem to be a promising scheme. Another method, used by Coppersmith, Lee, and Allmen uses a combination of silicon and silicon germanium to “stretch” the silicon layer, providing a more stable environment for the electron qubits. The advantage to the system is decreased error caused by interaction among the electrons.

The appealing aspect of semiconductor quantum computers is the ease of measuring and manipulating the qubits' states. Since the electrons used are in stable locations, unlike using qubits, they will remain there until otherwise moved. Petta *et al.* demonstrate a system to measure and change electrons' states in a semiconducting quantum computer, and similar to other articles, show that the error is slow to take effect. Another large advantage of semiconductors is that there is a working single electron transistor, outlined in the work by Conrad, Greentree, and Hollenberg, which encourages scalability as a possible result of work with quantum computers using a silicon semiconductor.

Many aspects inherent to semiconductors are beneficial to their application to quantum computers. The lack of magnetism at low temperatures is ideal for the use of either spin or charge to be used as the qubit in single or double quantum dot architecture, and that aspect is not generally found outside of semiconductors. However, nonmagnetic semiconductors are very expensive even in small amounts, so they are somewhat prohibitive in terms of the feasibility of increasing the scale of computers using silicon chips. Xue's solution, to use only singular quantum chips in otherwise classical computers, may provide a solution to the increased price. Otherwise, these systems will likely be limited to only medium-to-large companies and other wealthy parties looking to use quantum computers. They will theoretically scale to even an arbitrary degree as described by the relatively low error and application of the threshold theorem. Therefore, semiconducting schemes are among the top candidates for research in order to increase efficiency and decrease both price and computational requirements.

Scalability of semiconductors has been demonstrated in a quantitative analysis by Hornibrook *et al.* in which a supercooled semiconductor uses a field-programmable gate array (FPGA) to manipulate a qubit. The result of the demonstration is an application of a classical control scheme that is readily applied to quantum computers. Such a system would be extremely desirable: based on the threshold theorem, this would imply that scalable computers are not only possible, but relatively easily created. Using systems already available in classical computers would allow physical computers to be created using only small adjustments, and the macroscopic system would essentially just be a classical computer with its constituent parts being quantum. As classical computer construction is a well-developed field, expanding such computers would be trivial.

While the theoretical aspects of semiconducting quantum computation schemes have been addressed in research by Hornibrook *et al.*, separate problems arise in the availability of resources involved, and the conditions in which semiconductors work. In general, the silicon that is specially treated would be of a prohibitive cost for mass production for commercial or wide-scale consumer use. The majority of experiments use this treated silicon for its magnetic properties. Its magnetism is inherently insignificant, and therefore it does not interact undesirably with any quantum information. Because quantum computers fall apart when any notable interaction takes place within the computer itself (in this case the silicon) or the environment, using anything less than treated silicon would likely cause usable information to be lost.

The second physical issue with semiconductors as quantum computers is that most experimental demonstrations of semiconducting architectures have been done only under extremely low temperatures, usually far below 1 Kelvin. This is restrictive if it must be done for larger systems, since supercooling objects means providing a constant supply of coolant. Because scaling quantum computers must address the issue of losing quantum information and also the costs involved in physical production of such computers, using limiting structures like supercooled computers restricts possibilities severely. How far a quantum computer might go in its utility and ability to be recreated for commercial or personal use must also be addressed, and for this reason, semiconductor solutions at or near room temperature is an area that needs significantly more attention. Otherwise, quantum computers will only be of very limited application, and only used by wealthy companies or governments.

Single Dot Schemes

In Si quantum computers, the two primary possibilities researched are single and double quantum dots. While quantum dots are used for a single qubit regardless of whether they use single or double dots, the stability of the resulting computer depends substantially on which system is used. Calderón, Koiller, and Das Sarma discuss the possibility of a single spin system with individual electrons as its qubits. In their results, the researchers found that the spin is much harder to measure for a single electron than a double dot. Further, they state that "Double donors (S, Se, T) in Si are substitutional deep centers whose electrons' binding energies … are typically one order of magnitude larger than for single donors (P, As, Sb)." What this implies is that the quantum information stored in the double-electron system is much more sensitive, so results will theoretically be on the order of 10 times more accurate, thereby reducing error. Given this, since double electron donors are still very commonly used elements, including sulfur which is mentioned as an option, double donors are a much better option for scalable quantum computers than single counterparts.

One possibility that redeems the feasibility of using single quantum dots is outlined by Conrad, Greentree, and Hollenberg. Their experiment uses a Single Electron Transistor, or SET, to both measure and control the state of a qubit in a semiconducting quantum computer. While using a SET is not limited to single electron systems, the SET itself does use a single quantum dot to measure electric current. The issues outlined in other papers regarding sensitivity of single dot systems are not addressed. However, the ideas presented in this paper have two promising implications. First, the system is able to control the state of a quantum dot as well as measure the state it is in. This twofold application makes it very efficient, an aspect necessary in addressing scalability. Second, because the system is highly sensitive, the SET will have a relatively small margin of error on its measurements. Effectively, fewer error correction gates must follow a SET measurement, again decreasing the space requirement for SETs. So while the quantum dots themselves will likely not use single electrons as qubits in a scaled system, the single dot system still has possible applications.

Phosphorous electron donors have also been used as a possible form for single-electron semiconducting quantum computers. Pla *et al*. devise a manipulation and measurement scheme available for use in single quantum dot systems. According to the authors, "…it is a formidable challenge to combine the electrical measurement capabilities of engineered nanostructures with the benefits inherent to atomic spin qubits," so rather than attempt to reconcile the extremes of measurement of qubits based on spin states, they use the simpler methods that could be measured using SETs. Even without using specifically engineered samples of the semiconducting material, the study ended with results of 90% or greater success, much of which would be addressed with more specific supplies (specifically enriched Si substrates). The results demonstrated by Pla *et al*. indicate that, while double quantum dots have more backing in terms of research and technical results, the advanced state of single electron systems indicates they are possible and worth pursuing.

Double Dot Schemes

Petta *et al*. demonstrate a physical example of how semiconductors using double quantum dots might be physically realized. The system is created using GaAs/AlGaAs to create a 2-well system in which two entangled electrons of independent states rest. As mentioned previously, such a system has a high accuracy in measurement, and the SET would act as a possible method of measurement at the end of the computation. Petta notes that "This two-electron spin qubit may provide a starting point for implementation of quantum computation schemes with considerable practical advantages: [a]ll operations for preparing, protecting, and measuring entangled electron spins can be implemented by local electrostatic gate control," which is a well-developed area that could easily be scaled. Since the experiment's results demonstrate necessary attributes for scaling, the GaAs/AlGaAs system must be considered a strong possibility. It meets the requirements including entanglement, long coherence time, and high accuracy of measurement. While the same issues in economic expense apply here, the efficiency of the system reduces the costs associated with its production.

A GaAs/AlGaAs computer is not the only scheme available for double dot quantum computers. Coppersmith, Lee, and Allmen offer Si/SiGe as a viable option for a semiconducting computer. One of the primary issues addressed in their research is interaction between electrons in wells that are close together, which causes coherent error in any silicon-based system. Normally, though, increasing in scale will cause the coherent error to increase while the computer increases in scale. The Si/SiGe does not have this issue however, and according to their results, the system is “robust even in the presence of gate potential fluctuations and imperfections such as quantum well width variations,” In other words, the system described does not lose information caused by the electrons themselves. These interactions in most articles are categorized with other coherent errors, but the fact that a Si/SiGe system essentially removes this as a problem places it ahead of other methods.

Some of the research published regarding double quantum dots is among the most important step toward complete implementation of scalable quantum computers. This can be seen in an article by Xue, who outlines the gates and physical systems required for every step of a computation using two entangled electrons. While others have tried this, Xue’s research indicates high values for coherence time, accuracy of retrieved results, and scaling.

Xue’s research has more implications than a schematic for a double dot computer: it also provides a system which hardware could use to implement the system onto a modern classical computer chip. The implications of his idea are great for the feasibility of scaling chips that are quantum in nature into systems that are already used in computer science. While others tend to create entirely new quantum computers to answer specific problems, Xue's chip would be used specifically for implementing Shor's algorithm to factor large numbers, and would depend on classical hardware that would not need to be developed separately from regular systems.

Double dot quantum computation schemes so far have proven to be the best in terms of space efficiency and accuracy of results. Since the materials to create them are expensive, this is especially important in respect to the feasibility of their scaling. Each article demonstrates that scalability is realistic in double dot systems, but also in semiconductors in general. Because there is precedent in creating physical realizations of scaled versions of both double and single dot quantum computers, each should be considered in feasibly developing scalable quantum computers.

Chapter 2

Quantitative Simulation of Quantum Computers

Despite their limited application, quantitative simulations of quantum systems run on classical computers drive advances in the creation of quantum computers. This is unsurprising, since the actual production of such computers is currently extremely expensive for no tangible return. Simulating systems, though, requires no actual quantum computer. However, the drawbacks are inherent in the fact that such computers are not quantum at all. They are instead very basic attempts at the mathematical processes behind quantum computations. Still, since creating a true quantum computer is economically unfeasible except in a few special cases presently, their results are the closest available to actual realization of quantum computers.

Currently, the problem facing quantitative approximations for quantum processes is that scaling cannot be attempted accurately with simple simulations. Researchers might be able to address how specific error forms affect a computation scheme, but the results tend to be inconsistent. So true assessment of the scaling aspects of quantum computers cannot be approached this way in general. Still, the proposal and analysis of scalable architectures is valuable in providing the framework for physical creation of quantum computers that theoretically scale well. One thing to note about simulations is that they are often used as confirmation that a form of computation scheme is worth attempting. If even a simulation is unsuccessful, that idea is not pursued further.

Most of the systems described using quantitative analysis do not fit perfectly into a single category of a quantum scheme. Since they are not experimentally realized, current architecture is not a limitation to simulations. So while few of the systems described in this section are perfect matches for the category they are in, generally they are analogous and would best be described by one well-developed scheme. In this section, all research will be split into the scheme whose experiments are most similar to the methods used.

Linear Optical Simulations

Both cluster states and linked states are possible setups of scaled optical computers. Cluster states depend on inputting a large number of qubits simultaneously, processing them all using the same algorithm, and finding average values among all qubits. Linked states, on the other hand, entangle successive photons with two other qubits, one “ahead” and one “behind”. The end result is a chain of entangled photonic qubits, and in theory spreads the error evenly among the qubits. The linked state system is implemented theoretically by Mor and Yoran. Though the scheme still uses linear optical photonic qubits, no other system has been created previously that utilizes entanglement linearly between the qubits. The primary results demonstrate only a polynomial number of required photons to demonstrate entanglement. Therefore, the system should scale well without exponential increase in error or required materials.

Wang, Yang, and Nori further develop the use of cluster state optical qubits in order to scale computers without insurmountable error. The researchers describe their combination of atomic and photonic systems,

> "Cluster states can be easily generated and stored with atoms, but it is difficult to perform measurements on atoms. In contrast, it is easy to perform measurements on photons, but it is difficult to store quantum states using photons. Thus, this hybrid proposal uses the best from atomic and photonic qubits, to provide robust one-way [quantum computing]. Namely, to generate and store cluster states in an atomic system, then transfer to photons the states that are subjected to measurements, and then perform single-qubit operations and measurements on photonic qubits."

These results have profound implications in addressing scalability of hybrid atomic/photonic qubits. In effect, the long storage of information using atomic structures is similar to application of a classical computer storage system. Unfortunately, one major possible source of induced error is in exchange between the two forms of quantum information carriers. Wang, Yang, and Nori do not address this possibility, so their conclusion for a scalable hybrid system may be insignificant in its impact.

Wei and Deng propose another architecture using photonic and atomic qubits. However, rather than transfer between the two systems, they propose the use of a linear optical quantum scheme for beginning the computation, using entangled photons for nearly the entire process. The single quantum dot would be used late in computation, only as an agent for the measurement of the states of the final qubits.

The issue addressed here is the difficulty in creating interaction between photons, which is fixed by interaction with the quantum dot. Scalability comes from the ability to use highly-entangled cluster states with an arbitrarily large number of optical qubits. Since cluster states are a well-developed system of scalable quantum computers, their ability to avoid and decrease coherent error can be taken advantage of in this system. In the end, a CNOT gate architecture is implemented that scales well with both the photonic and atomic qubits in the system.

In a separate article, Wei and Dang design a set of gates that would fit in the same system as their other research. These include CNOT, SWAP, square root of SWAP, and three-qubit Toffoli gates. Each of these is fundamentally required for a useful quantum computer, including one that has the ability to undergo Shor's algorithm. All previous results regarding scalability of the system still hold, since the cluster states are used alongside a single dot scheme. The most interesting part of the implementation of all those gates is that, despite no full-scale quantum computer existing using these components currently, the entirety of a scalable quantum computer is theoretically possible with only those fundamental pieces.

Single Dot Simulations

On a simpler basis, some quantitative research focuses specifically on simulating one aspect of a quantum computer. For example, Das Sarma and de Sousa address the issue of whether the coherence time available to single dot quantum computers is feasible for production of scalable computers. While previous results indicated the upper bound for the timeframe is less than a microsecond, their research demonstrates theoretically that a P donor with a Si or GaAs quantum dot is feasible for long coherence times. Despite the simple premise of their research, proof that a reasonably useful time for scalability of single quantum dot schemes is significant in encouraging research in their use.

In contrast to manipulating only small number of atoms in a semiconducting quantum computer, Burgarth *et al*. offer the possibility of using a large scheme, but only analyzing properties of the extreme ends of a spin chain. The plan is fundamentally different from the linear optical qubit chain offered previously. In this case, the intermediate qubits are not used in final calculations. Instead, the middle particles are essentially used to shield against uncertainties caused by the end qubits. So if a small magnetic field is caused by the spin of one end, the environmental effects will not have any noticeable impact on the other end.

In terms of application, this method may have issues when used in a scalable quantum computer. Significantly more space is required to have a chain between every pair of entangled qubits, and there is still an environmental effect, albeit a small one, by each of the intermediate particles. Further, the time required for information to travel from one end of the chain to the other may not be insignificant, and the authors do not address possible incoherent error caused by computation time. Overall, while the idea is interesting, there is a lack of numerical support for why a chain of semiconducting qubits is more beneficial to quantum computers than a single pair.

Zhang *et al*. use single dots as a formulation for cluster states, but only as a scalable set of qubits. In other words, they prepare a large number of single quantum dots in one location, and then act on each one independently by applying a radio wave with a specific frequency to change its phase. Three notable improvements can be found in this system. First, using unique qubits with virtually no interaction produces individual results with essentially separate calculations. Such a system means error can be reduced by increasing the number of qubits manipulated until the threshold theorem is satisfied. Second, there is little room for error in terms of the excited states of bound electrons, as they are held at low energy. Third, the system is simple, can be controlled easily using radio waves and is scalable in an obvious linear way. All that is required is a larger physical system.

"Data qubits" offer another unique possible solution to taking a small system and making it scalable. Demonstrated by O'Gorman *et al*., data qubits are used in conjunction with single dot quantum systems. While entangled semiconducting qubits are held at large separation, data qubits that are unentangled, are sent past the entangled qubits. The change in the test qubits'spin caused by the spin of the entangled qubits is retained by these data qubits, which are then measured to obtain the results of quantum computation.

Not only did the simulations and small experimental system find extremely high fidelity (>99.98%) among the qubits, but the system is also theoretically very fast given the exponential increase in speed when the scheme is made smaller. The main issue faced by this system is that, since the entangled photons are kept at larger distances than most system, finding ways to increase its speed or decrease its environmental decoherence are difficult. Still, the authors state, "The footprint of the required electronic components to measure a single donor spin in silicon is typically on the order of 200 x 200 nm^2 and is thus small enough to achieve qubit grid separations of D = 400nm." While this might not be made smaller, keeping it at a similar size would offer polynomially increasing time and space required, neither of which is a hard barrier for achieving scalability.

Double Dot Simulations

Fewer quantitative demonstrations of double quantum dots are performed than single quantum dots. This is likely the result of multiple factors. One possibility is that single dot systems are simpler to simulate classically while entanglement and interaction between two electrons does not need to be taken into account. Another is the ease with which a hybrid between linear optical and single quantum dot architectures can be combined. Since that combination formulates a significant body of the research in the field of scalability, it is likely that there is little reason to utilize double dot systems over single dot ones. However, double quantum dots have enough fundamental properties that lend themselves to scalable quantum computation that the likelihood of a method combining linear optical and double dot schemes must not be overlooked.

Jefferson *et al*. propose one of the only scalable systems using simulated double dot quantum computers. Several prospective methods for scaling are demonstrated. For example, they propose directly changing the magnitude of qubits' states during computation in order to correct errors, an idea that is not regularly addressed in quantum computers. Generally, results of qubit interaction and fidelity are accepted immediately as either success or failure by the end of an experiment. Another new idea provided in the same research utilizes, where the electron is brought individually to an enclosed region, passed through a lowered potential barrier, then measured by a SET. Though neither of these currently has the technology required to be feasible for a scaled quantum computer, since they have been proven in concept, they should be explored further.

Shor's Algorithm Simulations

Outside the context of the forms of quantum computation schemes discussed to this point, the first quantitative simulations worth discussing are those that interpret scaling of Shor's algorithm in general. One benefit to quantitative analysis is that quantum computers in general can be viewed in general in order to confirm that, as in our example, Shor's algorithm does not break down when extended to scalable systems. If it is the case that the quantum algorithms used cannot be extended arbitrarily, all physical architectures to realize those systems become moot. However, this fortunately does not appear to be the case, and so it would seem that quantum computers do have theoretical practical uses.

Two articles take Shor's algorithm and simulate two different forms of error inherent in quantum computers. The first of these, published by García-Mata, Frahm, and Shepelyansky, specifically addresses environmental errors, namely those created between qubits within the architecture. Two of their conclusions must be weighed. First, the researchers state that, "[t]he results show that…the algorithm becomes not operational while below the border the factorization can be performed. This border drops only polynomially with the logarithm of factorized number N." Since scalability is only ever restricted fundamentally by exponential growth in decoherence, polynomial restrictions are technically achievable, and in many cases insignificant. Second, the results indicate that the only way to facilitate scalable quantum computers that are able to factor very large numbers, including those used in cryptography, the error in quantum gates must be extremely small.

Fortunately, effects of error in quantum gate operations are addressed in an article by Guo, Long, and Sun. The two types of error found are systematic and random, and according to the results of their analysis, "…the effect of the systematic errors is to shift the positions of the peaks, whereas the random errors change the shape of the probability distribution." Guo, Long, and Sun go on to say that quantum error correction is essentially sufficient for correction of the systematic errror, meaning the only requirement to fix those is more qubits that go through an error correction process. However, random error appears to develop as the result of coherent error increasing over more time spent on the algorithm. Realistically, the only way then to reduce random error is to increase coherent times (which has been demonstrated in previous articles) or perform calculations more quickly, which is dependent again on the number of available qubits.

Simulation of Uncategorized Architectures

This section will contain a collection of interesting results that are not classified in any quantum computer scheme described so far in previous sections. First among these is an adiabatic quantum computer described by Ashhab, Johansson, and Nori. Adiabatic systems bring qubits down to their ground state, and keeps them as close to that state as possible during manipulation. Resulting directly from that process, since it is highly unlikely for quantum states to be interrupted by the environment, the computer encounters a low magnitude of error. Due to this, adiabatic computers are theoretically among the most feasible in scalability. If it is in fact possible to create a scalable quantum computer with entangled qubits all kept within small margins of their ground states, in an ideal system the result would be minimal loss of information.

Next, Beals *et al.* address the scalability of optical lattice structures. The basis of the optical lattice architecture is that atoms in a crystalline structure have certain normal modes with specific available energy levels. These can be used as qubits by storing information in the atoms themselves. Many problems with scaling the system come up immediately in the research. For example, forcing a particle into one of its two distinct states changes the physical property of that atom, shifting its position in the structure. Doing so leaves the qubit to interact and lose information to other nearby qubits. 2-dimensional systems might be operated on all at once, and in fact the article formulates a system using 10,000 qubits in parallel. However, the results make it clear that 3D systems with up to 1,000,000 qubits, though very desirable, would be difficult to act on or measure due to its dense physical structure. Also, the proximity between qubits in a 3D system would increase environmental decoherence, reducing its chances of scaling.

Universal quantum computers are the final system outside the standard quantum schemes. Since no experimental or theoretical framework for such a system has been created, Sau, Tewari, and Das Sarma describe a scalable quantum computer that utilizes what they refer to as “a semiconductor quantum wire network.” While no physical interpretation of the results has yet been created, the quantitative analysis shows great promises. The authors of the article state, “[o]ur schemes for deterministically generating two-qubit entanglement and arbitrary single-qubit phase gates establish the semiconductor wire network as a viable platform for universal quantum computation.” Not only do the results contain demonstrations that universal quantum systems are theoretically possible, they also end in a high limit for how well the computer would work with error, up to a maximum of 14% success, significantly above the standard .0001-.001 generally accepted as sufficient for the threshold theorem.

Of these three less developed methods for creation of a quantum computer, the most promising appears to be the adiabatic quantum computer. Optical lattice structures have significant resource overheads, and the principles behind its scalability depend on prohibitively low error in the environment and also interaction between qubits. Universal quantum computers, though shown to be theoretically feasible, still have no experimental realization in any way. Also, the standard questions of coherent and incoherent errors are not thoroughly addressed, and those generally represent the most difficult obstacles to overcome for a system to be considered theoretically scalable. The adiabatic scheme does not share the same shortcomings of the other options: physical analogues have been explored, and the error present in the system is fundamentally low. Even if the error is put into the system, nothing bars adiabatic systems from undergoing quantum error correction after computation.

Chapter 3

Experimental Results

Whereas quantitative analysis of theoretical quantum computation schemes forms the basis of most research in quantum computing, experimental results confirm whether such tests are physically valid and should be explored further. The end goal in creating scalable quantum computers is not to prove whether such a system could exist. In fact, this has already been done several times, which is clearly seen in previous sections, but with trivially and uselessly small calculations. Rather, the goal in researching quantum schemes is to create systems which will answer problems that are unsolvable with current technology. The problem most researchers focus on, Shor's algorithm, is used primarily for its simplicity and for the fact that the principle behind it is well understood, but the various applications of quantum computers will be addressed in later sections.

As outlined in previous sections, addressing whether a quantum computer is scalable depends on several aspects of the quantum system itself, such as degree of entanglement, suppression of error, and length of coherence time. On a more utilitarian level, the cost of materials, environmental conditions, and physical space required to create a large computer must also be taken into account. The computers created to this point have tended to rely on very expensive, large systems at ultra-cool temperatures to process simple calculations. For anybody to invest into scaled quantum computers, it must be relatively efficient, which is in no way the case right now.

Scalable realizations may be a long way off from production, but the combination of systems outlined in the quantitative section offer several realistic approaches to scaling quantum computers. Researchers have demonstrated in many different computation schemes, a scalable system is not only theoretically possible but also experimentally sound. Some possible methods are ensemble quantum computing, strong magnetic fields, linear optical systems, as well as both single and double dot schemes.

Unscaled Successes

Before quantum computers could be created on an increasing scale, researchers needed to demonstrate that a quantum computer is a physical possibility. One demonstration of a successful quantum computation came from Vandersypen *et al.*, who managed to create the first experimental quantum computer. The system used they is a nuclear magnetic resonance (NMR) with F and C molecules used as qubits, whose computational information is stored in the spins of their electrons. The results of the experiment are very significant: not only did the group create a working quantum computer, but they also did so with a coherence time of greater than .7 seconds, a relatively long time in terms of holding information without losing accuracy to the environment.

The next important breakthrough came from Lu *et al.* who used a computer to factor the number 15 with a computer that they proved is quantum in nature. Earlier creations of possible quantum computers were created, but Lu demonstrates the doubt in whether the systems in the original experiments were truly quantum. The criterion used to measure this is entanglement: Lu proves the photonic qubits involved are definitely entangled before a full experimental realization of Shor's algorithm. The results did more than required. 50% fidelity of results is the minimum threshold for particles to be considered entangled, and the results indicate 99% fidelity. This level in certainty that the computer is quantum with extremely accurate results acted as a proof-of-principal for optical quantum computers, and also any entangled system in general. While results published by Vandersypen *et al.* undoubtedly advanced quantum computation, in hindsight their study was flawed in its failure to address whether the computer they created was technically quantum instead of classical. Lu's results build on this significantly, demonstrating that the seemingly unapproachable idea does show physical feasibility, at least in principle. Effectively, Lu provided encouragement that quantum computation is a field worth the time and resources invested.

Scaled NMR Systems

Two major results in scalability have come from application of NMR as the basis for a quantum computer. The first, published by Cory *et al.,* uses NMR spectroscopy to apply searches to significantly more qubits per cycle than usual schemes. Through ensemble quantum computers, which utilize the average value over a large number of qubits that undergo the same computation, similarly to cluster states, the researchers obtain scaled results. In the article, Cory states, "We have described a macroscopic analogue of a [quantum computer] that can be implemented today, using commercially available NMR spectrometers and ordinary liquid samples," demonstrating that scaling in an NMR system is feasible.

However, problems occurring in the system detract from its scalability. Primary among these is the margin of error required to reach a conclusion in reasonable time. The article does not address solutions to avoiding significant incoherent error. The facts that the NMR scheme provided a correct solution and that the system is scalable in theory simply using greater sample sizes are both helpful in concluding that NMR spectroscopy is a legitimate possibility for scaling quantum computers.

The second interesting advancement in liquid-state NMR use is outlined by Lages and Shepelyansky. Their results depend on a large ensemble and the application of a strong magnetic field over the entire sample. Primarily, the article addresses a method in which information can be protected from the environment. That goal is realized through application of a strong magnetic field, which causes all fluctuations around the qubits and between pairs of the qubits themselves to be relatively insignificant, therefore reducing the error to negligible levels.

Lages and Shepelyansky are highly successful in suppressing all types of error. They state, "[The numerical and analytical studies] clearly show that a presence of magnetic field gradient allows suppressing quantum chaos in the quantum computer hardware if the gradient g is larger than the quantum chaos border…" Clearly, this result is significant in NMR spectroscopy: all the issues encountered but not addressed in Cory's research can be effectively ignored if some strong magnetic field is applied to the sample used. Combining these two sets of research indicates that NMR schemes have the feasibility to scale well, without overwhelming loss of information.

A unique application of an NMR computer can be seen in work done by Shankar, Hegde, and Mahesh. The system in their article uses the computer to simulate one-dimensional quantum systems. It has been proven that no classical computer has the technical capability to simulate systems that are quantum in nature, so reliance upon quantum computers is the only possibility. The end results of the simulations of Schrödinger's wave equation matched very well with values expected by the researchers, and also had no contradictions with theoretical descriptions of systems that include free particles as well as particles in certain potential fields.

Scaled Linear Optical Systems

In linear optical quantum computers, scaling faces unique challenges not addressed in NMR systems. While NMR depends on a large number of qubits being acted on all together and then later measured through varying means, optical quantum computers are built with the intent of single or double qubits being processed at a time. What this causes is a significant loss in the efficiency of the computer itself for a couple reasons. First, the increased time required for calculation makes the minimum coherence time barrier significantly higher. Therefore, a system must be found that either can suspend quantum states for longer lengths of time, or the computation itself must be made faster. Second, there is some discrepancy between error inherent in logic gates and qubit measurement success. So while two gates may each have 80% fidelities, a qubit that goes through both gates could still retain >99% success. That discrepancy is not present in NMR systems.

Lanyon *et al.* address both of the issues inherent in linear optical systems in depth. While their results do demonstrate an experimental success of Shor's algorithm, they also illustrate the shortcomings in attempting scaling of a linear optical quantum computer. One major advantage in comparison to NMR architecture is the knowledge that the system is quantum in nature, providing basis for a quantum computer with much greater certainty than with NMR spectroscopy.

According to Lanyon, gates used in optical schemes "do not require pre-existing entanglement and…encode our qubits into the polarization of up to four photons," which may directly be used in scaling the quantum computer. However, because there is no standard way to describe quantum gates using four inputs, it is currently impossible to see how the result of coherent error from a scalable gate will affect final outputs.

Using a different setup than a standard linear optical system, Choi *et al*. are able to use trapped ions as qubits in an optical quantum computer. As a direct result, unique possibilities for addressing scalability issues are discovered. Since trapped ions already have fundamentally low levels of coherent error, suppression of that error in a scaled quantum computer requires significantly less error correction. Also, the group demonstrates an architecture in which more than two entangled qubits could be addressed simultaneously, further expanding the possibility of scaling.

Some research indicates that linear optical systems could contain unexplored features that would greatly increase the feasibility of scalability for quantum computers in general. In fact, the features of said systems could theoretically be applied to non-optical schemes as well. Zhao *et al.* propose a blueprint in which information stored on qubits could be sent across long distances. The primary use of such a system would be sending information about a certain state from one part of a quantum computer to another part, so that information could be recycled for further use.

Zhao's proposed system has significant physical ramifications. Among these, the architecture has coherence times of greater than 1ms, 3 orders of magnitude greater than any other communication between gates. The next logical step from this is the ability to store mass quantum information, and in doing so, create a scaled quantum computer. Because information can be experimentally stored now in this manner, one linear optical system of modest size could be recycled, with each result of the computation being stored non-locally. That information could later be retrieved for further calculation. Applying a system in this way provides a new theoretical construction system for scalable linear optical quantum computers.

Scaled Single Dot Semiconductor Systems

Single dot semiconductors have significantly less experimental support than double dot systems. Most likely, this is caused by the beneficial and convenient properties inherent to double dot systems, such as the significantly greater magnitude of spin. Properties like this make working with double donor systems much easier. Still, some of the end results of experiments using single dot schemes have advanced quantum computing in general, via a combination of multiple schemes.

An article by Calderón, Koiller, and Das Sarma, in tandem with other previous experiments that allow for coherence times near a second, provide a possible solution to avoiding coherent error. The use of a single electron donor for isolated qubits has one interesting result: Calderón describes the unique state when he says "if they form a triplet, selection rules imply a much longer lived state." This unique situation does not come as a result of double donors, only single. By using only single dot triplets, it might be possible to combine the system with others because of its abnormally long coherence time and high accuracy.

As discussed, the use of single dot architecture could be combined in unique ways with linear optical schemes. Economou *et al*. support this idea, stating "[s]pins…have received a great deal of attention because they interact strongly with light and provide the opportunity for ultrafast all-optical implementation of logical operations." While the negative implications of that statement, especially in decoherence caused by environmental interaction, have been discussed, there are many potential benefits as well. Foremost among these is the possible combination of single dot semiconductors with linear optical systems. Since each of these has unique properties in terms of feasibility of scaling, it may be possible to combine the two methods in order to create a quantum computer that scales without the problems that arise in using one individual system.

Single electron donor systems like the one proposed by Economou have the potential to have scalability beyond that of other schemes. The group proposes two issues that need to be addressed in scaling: the ability to "tune" the system in order to prepare states for computation, and the difficulty in entangling arbitrary qubits. Since Shor's algorithm depends entirely on randomly-chosen states, if two qubits begin in incompatible states, the computer will fail. However, since the fidelity of states is relatively high in all circumstances, the research group determines that the threshold theorem is upheld and information obtained is more than accurate enough for a scalable system.

Scaled Double Dot Semiconductor Systems

Similar results to those discovered in single dot systems have been developed for double dot quantum computers. A prime example of this is seen in the article by Petta *et al*. who, using photons to control qubits' phases, increases coherence time of double dots. While this is nothing entirely new in quantum computing in general, extending this result to double dot systems means similar methods combining multiple methods might be applied also to double quantum dots. Once again, if a scalable quantum computer is ever to be realized, if the limiting aspects required in reduction of various errors can be covered by a combination of quantum computation schemes, then it is worth exploring all possible options. It is possible that in the future, the combination of systems will be what allows for scaled computers, rather than one specific method.

By introducing another method using double quantum dots, Veldhorst *et al.* create a quantum computer using a combination of two methods. The first of course is a double quantum dot, using two electrons in a double potential well. The second, rather than analyzing the qubits based on their charge distributions, uses spin of both electrons. This results in very long coherence time, but also causes measurement and entanglement to become more complex. According to Veldhorst *et al.*,

> "Here, by realizing a quantum dot qubit in isotopically enriched Silicon (28Si), we…show that all of the above coherence times can be improved by orders of magnitude. These long coherence times…lead to low control error rates and the high fidelities that will be required for large-scale, fault tolerant quantum computing."

Original solutions like this show promise in the development of quantum computers into the realm of scalable. By combining long coherence times and high accuracy, the article demonstrates how fault tolerant systems could exist in physical terms.

Combined Scalable Systems

Few articles specifically approach the idea of combining multiple quantum computation schemes into one quantum computer. In general, as can be seen in previous references, they focus on one of the most common qubit structures or macro architectures. Linear optical systems, NMR computers, and single and double quantum dots are all thoroughly explored. In most cases, the limiting parts of those systems are explained and an answer is given regarding whether or not further development is warranted.

Analyzing the scalability of certain setups always comes down to checking several specific traits. Computation and coherence time must be low enough to avoid loss of information; accuracy of results must be great enough to pass the threshold theorem; and qubits must be sufficiently entangled so the computations remain quantum in nature. If any of these is not met, or breaks down when trying to increase the degree to which the computer works, the system will not scale.

While each physical, experimental system has failed in some aspect of scaling, each also has some part that offers an advantage over other systems. For linear optical architecture, availability of photonic qubits and a high degree of entanglement are present. NMR systems offer very easy macroscopic results for measurement with expected values. Quantum dots have the potential for very high coherence times with relatively low errors.

The final development referenced previously had significant success through combining multiple systems into one cohesive quantum computation scheme. This is a unique approach to creating quantum computers, and one that appears to have more promise than any single system used in an isolated computer. Research by both Veldhorst *et al.* and Petta *et al.* supports the conclusion that the most effective way to formulate a scalable computer is through a combination of multiple methods.

Overall, it is a stretch to consider anything more than the most basic quantum computers to be experimental successes in terms of scalable quantum computing. The lowest forms, which factored trivial values, were only significant as proof-of-principle for quantum computers in general. Focus must now be placed on whether these ideas can be extended to realistically larger systems with more widespread applications.

Chapter 4

Minimizing Quantum Error

Correcting for quantum errors takes multiple forms, corresponding to the various possible types of error in the system. Most commonly, error correction schemes are implemented at the end of manipulation of qubits, thereby reducing coherent error. The other forms of error, including environmental decoherence and random error, cannot be addressed as directly as coherent error simply because they generally are unpredictable. Addressing these must be done through indirect means, such as decreasing computation time, decreasing the error inherent in gate operations, or repeating computations and taking the average so errors are minimized.

Among the many reasons for focusing on systematic errors is the fact that they are easily fixed. In comparison to environmental error, caused by uncertain aspects of the system itself, the coherent error is known prior to computation, making it easy to create gates specifically to readjust qubits. Also, scaled systems that use error correction throughout the process are guaranteed to have more accurate results, increasing the probability of obtaining correct values. Unfortunately, other forms of error still propose significant issues in creating scaled quantum computers.

This section will be split into architectures used to correct systematic errors and those used for incoherent error. There is about as much research put into systematic error as all other forms combined, and coherent error correction schemes have been used in realistic situations, including some outlined in the experimental chapter. The variety of options available for coherent error indicates the extent that field covers, so some methods will be addressed that use indirect methods in order to avoid error. The difference in those cases is that instead of correcting for error that has already been introduced into the system, in some cases decoherence is just avoided entirely by making the system more efficient in time or in the number of qubits used in computation.

Scaling of Coherent Error Correction

Prospects for quantum error correction in working experimental quantum computers are positive. Coherent manipulation of qubits' states is made possible by knowledge of the unitary operations applied originally on them. Quantum algorithms depend on states that are by nature "messy", in that quantum mechanical properties of the qubits involved make it impossible to know exact information about the qubits. So error correction is an integral part to creating scaled quantum computers, as there is no feasible way in which an error-free computer could be created as per the threshold theorem.

Reichardt and Grover expand on this issue by arguing that coherent error must be addressed above all other errors, since once a computer is scaled, all error intrinsic to the computation gates will likewise scale. If the algorithms to suppress that error are inefficient, they will only become more cumbersome as the computations and computer become larger. Reichardt and Grover compose a sequence of what they call composite pulses used to counteract all the available types of coherent error. Each pulse acts on the qubit, undoing any error caused by the computation at a quantum gate. In their own words, "Therefore this composite pulse sequence allows for an arbitrarily accurate set of universal gates, giving a threshold result for this error model." By answering the requirements set forth by the threshold theorem, and by decreasing coherent error efficiently to an arbitrary level, the researchers manage to define a fault tolerant error correction scheme.

Due to the low time required for computation in any quantum scheme, time scaling is not generally an issue as quantum computers by definition are exponentially faster than classical computers. The direct result of this is that if error correction increases computation time, but decreases the number of qubits or space required to produce its desired output, then it sacrifices little for a substantial benefit. Martín-López *et al.* take advantage of this fact by reducing the qubit requirement for a quantum error correction architecture. They state,

"So for full scale implementations, qubit recycling reduces the total number of qubits required from [3log(2)N] to [log(2)N]+1; the only penalty is a polynomial increase in computation time, while the exponential speedup is retained-*i.e.* it is scalable. In general, saving in qubits can be more than 2/3 if more control qubits are required, or less than 2/3 in smaller proof of principle demonstrations such as this."

After implementing a CNOT gate followed by the error correction gate, Martín-López *et al.* obtain a successful value of 99% and an error margin of 4%. Since this is well above the requirement set by the threshold theorem, these results demonstrate that the system is technically scalable.

Unlike standard quantum error correction schemes, Qu *et al.* implement the idea of addressing coherent error numerically post-computation rather than using unitary transformations. This change represents a significant branch away from the standard system. Normally, since the operator acting on a qubit is known prior to processing, near-unitary operations have similar decoding procedures. Using properties of linear algebra, undoing the error is a simple process, though not one without overhead. Qu *et al.* offer the possibility that the method used to avoid or repair incoherent error should be applied to coherent error as well.

Among the most advantageous parts of this is the ability to avoid additional qubits only for error correction. Because in many cases experimental quantum computers use more qubits for error correction than quantum gates, coherent error correction is very inefficient under that method. Whether this has significance after any quantum architecture is scaled is a possible problem. While the decrease in qubits for error correction is beneficial, once a system is scaled, the number of required qubits will still increase exponentially, regardless of whether there are more qubits used for error correction. Qu *et al.* only address the simplicity of procedures for obtaining the information and simplifying calculation. They do not analyze whether or not the results will remain important once an experimental quantum computer becomes significantly larger.

Two important results come from analysis of coherent error correction schemes. One of these is the ease of scalability. Since quantum computers' power scales exponentially with size, time can be sacrificed for resource requirements or error correction without loss of information, success rates, or scaling. This is not an insignificant result: retaining the ability to make systems more physically efficient while ignoring time scaling leaves more options for possible physical realizations. The other primary result of coherent quantum error correction is the idea that error is intrinsic to quantum gates. While this would appear to be a direct extension of quantum mechanical rules, the analogue between quantum errors and error correction gates is surprising. Reichardt and Grover's work in finding the best way to address systematic error over large scales attests to the complexity of error correction, and encourages pursuing other solutions in that field.

Indirect Error Correction

Most quantum computational error comes in forms that are uncorrectable through means of shifting phases that coherent error takes advantage of. Because of this, the unknown nature of the errors must be accounted for by avoiding error in the first place, or by taking a more general approach of increasing system sizes so errors are less significant in the final results. The latter has severe limitations, since the size of the computer might be increased exponentially to accomplish this goal. Doing so would of course go against the goal of scalable quantum computers, which aim to produce useful schemes that only increase polynomially with computation size.

Fowler offers the first solution to creation of a scalable, computationally efficient quantum computer. Without limiting the type of quantum computer to any specific scheme, Fowler manages to reduce error by exponentially speeding up the computation time. In his analysis of the results, Fowler states, “This enables fault-tolerant quantum computation to be performed orders of magnitude faster than previously thought possible, with the execution time independent of the error correction strength.” The most important part of that statement is the last phrase; since the overall execution of the error correction scheme does not scale with increased error, it will be constant regardless of how big a computer is and how long execution takes.

Because Fowler’s work depends on an error correction scheme that works randomly, all possible error strengths are addressed with equal likelihood. In a way, his scheme itself simulates a quantum computer, searching for any feasible solution and adjusting for it if an unsuccessful state is found. Because of that process, Fowler states, “almost all known fault-tolerant universal quantum computation enabling [quantum error correcting] codes…achieve time-optimal quantum computation with modest additional quantum circuitry.” This fits very well with the goals of scalable quantum computers: by increasing their size, error correction itself does not need to be increased by any significant amount. Fowler’s results also demonstrate a quantitative simulation of finding factors of large numbers, granting credibility to his article. The results indicate that error detection takes time in the same order of magnitude as computation, meaning it is not exponentially larger, and therefore it scales.

Using gates that correct errors dynamically, during rather than after computation, allows Khodjasteh and Viola to create a robust method to removing multiple types of error in scalable systems. While the standard error per gate required for scalable computers is .001 as defined by the threshold theorem, the article addresses a system that would have this property in order to prove it is scalable. The authors state, “While our present construction addresses arbitrary linear decoherence, different algebraic error structures may be tackled by modifying the [dynamic decoupling] group.” Because the discovered error is quadratic (and therefore polynomial), and because the errors per gate in the proposed scheme do not increase with coherent error, it appears all incoherent error is corrected in the given architecture. Like Huang and Wei’s system, combining a dynamic error correction scheme in with a proven coherent error correction scheme could lead to a full, robust error correction scheme.

In one of the only experimental realizations of a robust quantum error correction method, Pudenz *et al.* describe an experimentally scaled version of an error corrected annealing quantum computer. While discussing the results, they state, “We demonstrate a substantial improvement over the performance of the processors in the absence of error correction. These results pave a path toward large scale noise-protected adiabatic quantum optimization devices."

Though it might fall under the category of coherent error correction, Weinstein introduces the option of decreasing the occurrence of error correction gates inside any given quantum computer. So instead of applying an error correction after every quantum gate in the computer, Weinstein proposes removing any gate that is unnecessary. Since the coherent error introduced by a gate is known prior to computation, changing the distribution of error correction gates should have no risk of causing information loss. Results of the decrease in gates decreases the qubit and computation time requirements, but those results do not scale exponentially. No numeric example used in the research has a qubit reduction of even 50%, and the stated impacts do not greatly affect whether exponentially scaled quantum computers are feasible. Despite this, if scalable computers are in fact possible, the reduction would still decrease computation time significantly.

The only commonality among the methods to avoid various types of decoherence is the fact that each takes advantage of some scalable feature of quantum computers. They might be made more efficient, have aspects of the computation switched or run concurrently, or even have the error itself be analyzed as if it were a quantum system. Since not one of the systems is similar in method to any other, none of the possibilities can be discarded until more research has been done on whether the results might be extended to scalable quantum architecture. Each method affects a different part of the evolution of a qubit through a calculation, so in theory it is also possible that some of the methods could be combined. Regardless of the end result, the possibilities outlined throughout the section offer proven ways to simplify and increase efficiency of a quantum computer.

Combined Error Correction

Some of the greatest successes in error correction come as the result of using two distinct schemes and combining them. Whether that means two error correction methods and applying them together, or using two forms of qubits that transfer information between each other, the use of multiple methods helps avoid overwhelming issues in scaling. For example, using photonic qubits for computations causes fast calculation time, then transferring the information to spin qubits stores the information while other data is being processed by the computer. Such combinations offer more possibilities than a quantum error correction scheme that is limited to one form.

Because error correction depends on retaining information while measuring the qubit's state and obtaining the final output, coherence must last long enough for the quantum computer to correct the qubit's state during error correction itself. Many systems would benefit from a system that had longer coherence time, since then errors could be corrected without the risk of decoherence mid-computation. Waldherr *et al*. create a computer in which electron spin, in combination with semiconducting qubits, corrects for error over longer times. So the benefits available in using hyperfine interaction as a measurement method for the semiconducting qubits can be retained while also using spin to store information, which lasts longer and therefore has a lower chance of information loss.

Increasing the number of parallel quantum circuits running at a time is another method of decreasing decoherence introduced by Huang and Wei. Using a universal quantum computer and cluster states, the researchers form a more efficient quantum computer. They develop proofs that lead to the conclusion that if cluster states are used, the computer can be scaled infinitely with no increased overhead in time or qubits required.

Huang and Wei's conclusion has two possible shortcomings. Since the errors corrected for do not include single qubit error, those must be corrected for separately, likely through another standard error correction scheme. Also, because computational systems are produced in this method in parallel, increasing the scale of the computer means producing a larger computer. Fortunately, the authors demonstrate that the size only increases polynomially, but since it must be produced in conjunction with coherent error correction architectures, it may not be an efficient architecture. In spite of this, the system does theoretically scale, and sufficiently answers the question of how incoherent error might be reduced.

Kosut, Shabani and Lidar offer a solution to the problem of insufficient robustness of coherent error correction. While generally only specific forms of error such as bit flips or sign flips can be fixed in coherent schemes, all other forms of error are left unaddressed. Adjusting for those errors, according to Kosut, Shabani, and Lidar, requires exponentially increasing amounts of time to correct if applied to a quantum computer. So rather than use limited schemes, the article illustrates a set of fault tolerant formats for addressing incoherent error.

The researchers note that, "The [computational] cost can vary greatly if the algorithm is modified for the specific problem structure." What this indicates is that the architecture will perform less efficiently if nothing is known about the system initially. However, the results of the study find that several forms of error are easily addressed using a relatively low number of qubits. Also, the results are scalable, since the form of the error is not assumed at any point, meaning the system is robust in detecting error both from the environment and from quantum gates.

Omkar, Srikanth, and Banerjee offer an original description of quantum computers by means of quantum error correction. The system used is essentially defining a quantum computer based on the error correction codes required to bring it to maximum possible certainty. Though the system is a significant departure from standard methods of quantum computation descriptions, it offers a significant advantage. Since the authors use their system to categorize quantum computation schemes based also on environmental, incoherent error, that error can be addressed without needing to avoid slow computation times. While a system is fully described, each aspect of it is known before computation, so error correction gates can be implemented with little uncertainty, effectively allowing scalability. According to Omkar, Srikanth, and Banerjee, “It can better cope with noise that changes over time-scales smaller than that required for a full characterization of the noise. This makes it well suited for real-time applications like feedback control of open quantum information processing systems.”

After describing the many possible systems used to correct for both coherent and incoherent error, it is easily seen that no single system dominates the body of research in robust error correction. Results include combining intrinsic properties from multiple systems, increasing the number of error corrections going on at once, and even using as few gates as possible to speed up the process. Out of all of this, it is most obvious that before a scalable quantum computer can be feasibly produced, new developments must also be made in the field of robust incoherent quantum error correction. The coherent errors, which will be discussed in the next section, are in comparison much easier to repair, and scalability of coherent error correction schemes is what must be taken into account.

Chapter 5

Extensions and Applications of Quantum Systems

As quantum information processing has developed as a field of study, numerous approaches to previously unsolvable problems have arisen. Some of those applications, such as Shor's algorithm, are already strongly developed and the results are commonly known. Others, like simulation of physical quantum systems, are only explored in theory and with small simulations, taking a less prominent spot in the field than Shor's algorithm or Grover's algorithm. In addition, though it has been slow to develop, mathematicians have also begun to develop a system supporting quantum logical systems.

Quantum Logic and Quantum Computation Theory

Due to the fact that quantum computers are intrinsically different from their classical counterparts, a new formulation of programming and logic schemes must be realized before scalable quantum computers have any real purpose. Otherwise, any computer that is created will be limited only to one specific problem. So while it may be useful to have a computer that finds the factors of one specific number, that solution is expensive and will not lead to the development of quantum systems overall.

Before a quantum programming language can be developed, there must be mathematical proof that quantum logic is inherently different from classical logic used in standard computers. Dunn *et al.* develop this proof by defining a set of variables in a quantum system, and show that classical logic is a subset of the simplest form of quantum logic. They then go on to define quantum systems of higher orders, and prove that each higher order is greater than all sets below it. One of the strange results of these proofs is that, given an arbitrarily high-dimensional quantum logic system, the power of quantum computers is without a theoretical bound. This idea implies that quantum computers are not only more powerful than classical computers in all cases, but also could be made the most powerful form of computer possible.

The second necessary development in creating scalable quantum computers focuses on the threshold theorem. While the original theorem depends on strict definitions and ideal circumstances, the reality of quantum computers depends on noisy systems, error correction, and decoherence. Dyakonov redefines the issues surrounding definition of errors, because the threshold theorem makes many assumptions using a mathematically perfect computer. By describing the issues with assuming a mathematically ideal interpretation of the physical world, Dyakonov shows why the threshold theorem holds little weight in whether scalable quantum computers are feasible or even possible. According to his results, it is better to address scalability of quantum computers not by whether or not they abide by the threshold theorem, but rather on whether physical implementations of a quantum scheme actually work when made experimentally.

Since now it is clear that quantum logic is inherently powerful and distinct from classical logic, classical code is insufficient for taking advantage of the extent of a quantum system's application. Quipper, a programmable language developed by Green *et al.* provides a unique and straightforward solution to the gap in quantum logic. Quipper is a system in which a classical computer is applied to analyzing the states of a quantum computer. Because the inputs and outputs of quantum computers are still generally classically described, such as numbers in Shor's algorithm, operating under a classical framework does not produce problems in interacting with a quantum computer.

Most importantly among the system put in place with Quipper is that the logic can be applied even to scalable quantum systems. As a result of the classical inputs and outputs, the only requirement for a scalable quantum computer language is that it must not introduce insurmountable error, in order to satisfy the threshold theorem. This is the case in Quipper. Green *et al*. state,

> “We demonstrate its usability to implementing seven non-trivial quantum algorithms, chosen to represent a broad range of quantum computing capabilities…Programming the seven algorithms required approximately 55 man months and resulted in a representation usable for resource estimation using realistic problem sizes. On this basis, we conclude that Quipper is both useable and useful.”

These results are significant, as Green *et al*. produce a physical interpretation of quantum results. This demonstrates very clearly that quantum computers can be manipulated using classical computers, making their results easy to interpret using algorithms and computational understanding available now.

Quantum Computer Applications

Many interesting but classically computationally difficult problems are efficiently simulated by quantum computers. Take, for example, simulation of physical quantum mechanical problems. Sornborger creates a quantum computer that simulates electron tunneling under standard quantum mechanical properties. While a working simulation of the system on a classical computer would take exponentially longer as size increases, it only takes polynomially longer on a quantum computer. Sornborger's system scales with $\log_2 N$ qubits where N particles are simulated. So in comparison to standard techniques used to see the development of a tunneling electron, this one requires few qubits and scales indefinitely.

Another well understood problem that is difficult to model on a classical computer is the many-body problem. The interactions between particles develop in complex ways due to their quantum mechanical properties, and so no good approximation of their trajectories is feasible using only classical computer methods. However, Alarcón *et al*. outline a physical interpretation of a quantum computer that could simulate the many-body problem. Because the researchers use particles' Hamiltonians to analyze their trajectories, these are well defined systems. The system was quickly proven in principle by the fact that a two-particle system, which can be developed classically, is exactly found when using quantum computation.

Classical and Quantum Cryptography

Because quantum computing developed in large part by the problem of factoring large numbers, there are numerous questions about the safety of classically encrypted data with scalable quantum computers. Shor's algorithm, by efficiently factoring those encryption keys, can break any modern system using prime factorization as its basis.

While there is worry for whether quantum computers will break all encryption protocol used, there is also substantial research into quantum encryption systems that are as secure with quantum computers as current protocol is with classical computers. That is to say, efficiently breaking the encryption would take an extremely long time even using quantum computers. For example, Marshall and Weedbrook create a quantum key distribution system which does not even assume anything about the error present in the quantum computers of either person using the key. While the authors state that previous key distribution methods assume near-perfect control over the error of a computer (similar to the threshold theorem), their system requires no such assumptions.

Their solution, rather than use discrete variables as keys, uses continuous variables. The difference lies in the fact that if the correct key requires a certain value, any error creates a non-zero probability that the keys will not be found correctly. However, by using continuous variables, an infinite number of possible values would be valid, allowing for a certain margin of error in either party's quantum computer. This method of a fault tolerant key distribution scheme is promising for protecting information from quantum computers.

While many other solutions exist that demonstrate the possibility of a protected quantum encryption system, this one example is sufficient as a proof of principle. Other solutions would be redundant, as the fact that cryptography as a whole is still protected under scalable quantum computation schemes is enough to cover that worry of security. Shor's algorithm, which drove the early development of quantum computers, introduced significant worry in whether creating such powerful computers is ethical. Fortunately, the research done in new systems demonstrates that there is no worry about whether unbreakable systems will exist after the mass use of quantum computers.

Conclusion

The research surrounding all the architectures and theory behind quantum computers is vast and in a lot of cases contradictory. As the field is still young, the rules change often and what is considered an insurmountable problem now might be trivial to solve for in a year. This has been the course so far in quantum computing. In the earlier stages, there was significant doubt into whether a quantum computer could be created at all. Now researchers are asking how big of a system can be created. Some articles depend entirely on the threshold theorem to prove results are scalable, while others state that the theorem is insignificant and does not lend credibility to scalability. Different quantum computation architectures get more or less focus depending on whether similar methods have produced promising results in the recent past. In the end, the field is still advancing rapidly, with entirely new results published often.

Of all the feasible systems for incoherent error correction, robust schemes offer the greatest theoretical potential for avoiding all types of decoherence. While performing parallel computations or decreasing computation time might be beneficial in making a quantum computer faster, when discussing extremely large computers, those small differences are unimportant. There should be a much greater focus on the errors that scale exponentially along with the system, rather than decoherence over time, since that can be adjusted for with much less effort. Among the solutions for robust error correction, some of the most promising are the ones that work universally. Since it is not yet known what form of quantum computer will dominate in scaling, the error correction methods that are limited to a single form are of lower priority than the ones that are universal.

Coherent error correction offers much less of an issue with the creation of scalable architecture. In part, this is because coherent error correction has already been implemented into successful experimental quantum computers, so it is accepted that they work as intended. Also, since these errors are always of forms that are approximately unitary, how they impact the system is known even before computation. So scaling will become less of an issue of how to decrease coherent error and more of a focus on how many error correction gates would be ideal. Still, coherent quantum error correction does represent a large portion of the qubits required in a quantum computer. So once the computers are scaled, a lot of the overhead in physically building it will depend on how efficient the correction scheme for these errors is. Fortunately, though, coherent errors do not ultimately limit whether or not a scalable quantum computer is feasible.

In terms of the type of what quantum computer is likely to be scaled successfully first, the types with the most momentum and promise are linear optical and double quantum dot systems. Both have their downfalls when it comes to how they address error, and how they could be scaled without becoming very expensive. However, each of them has already been shown to have the ability to scale to an arbitrary degree, and each has also been realized experimentally in small systems. Outside of those two possible candidates, NMR and single quantum dot architectures have some possibility of being used, but have lost favor because of error correction problems. Still, some of the more obscure computation schemes, especially those utilizing more than one system, might offer the most positive option overall. This could change relatively easily since combining multiple systems has a reasonable possibility of retaining the most useful properties of each.

Computation is on the edge of a redefinition. Quantum computers have already been proven, and though they are not yet developed, neither were classical computers when those came into being. Incoherent error has more and more possible solutions coming in, and coherent errors are being discussed in terms of efficiency instead of whether or not they can be done at all. There have been many successes in creating computers that are truly quantum in nature, of such a variety that it is likely at least one is scalable. Most importantly, every one of the results shows that scaling is possible within reasonable restrictions, all of which can be realistically met. There is every reason to believe that useful, scalable quantum computers are feasible.

Works Cited

Alarcón, A. et al. “Computation of Many-Particle Quantum Trajectories with Exchange Interaction: Application to the Simulation of Nanoelectronic Devices.” arxiv.org. 2 August 2014. Cornell University Library. 18 October 2014. http://arxiv.org/pdf/1408.0388.pdf

Ashhab, S., J.R. Johansson, and Franco Nori. “Decoherence in a Scalable Adiabatic Quantum Computer.” arxiv.org. 22 November 2006. Cornell University Library. 15 August 2014. http://arxiv.org/pdf/quant-ph/0608212.pdf

Beals, T.R., J. Vala, and K.B. Whaley. “Scalability of Quantum Computation with Addressable Optical Lattices.” arxiv.org. 11 April 2008. Cornell University Library. 12 July 2014. http://arxiv.org/pdf/0804.1971.pdf

Burgarth, Daniel et al. “Scalable Quantum Computation via Local Control of Only Two Qubits.” 29 June 2009. arxiv.org. Cornell University Library. 23 January 2014. http://arxiv.org/pdf/0905.3373.pdf

Calderón, M. J., B. Koiller, and S. Das Sarma. "Single Electron Spin and its Coherence in Si Quantum Computer Architecture." arxiv.org. 19 May 2013. Cornell University Library. 11 October 2013. http://arxiv.org/pdf/cond-mat/0610089v1.pdf

Choi, T. et al. “Optimal Quantum Control of Multi-Mode Couplings between Trapped Ion Qubits for Scalable Entanglement.” arxiv.org. 8 January 2014. Cornell University Library. 15 October 2014. http://arxiv.org/pdf/1401.1575.pdf

Conrad, Vincent I., Andrew D. Greentree, and Lloyd C. L. Hollenberg. “Multiplexing Single

Electron Transistors for Application in Scalable Solid-State Quantum Computing." arxiv.org. 6 February 2008. Cornell University Library. 20 October 2013. http://arxiv.org/pdf/cond-mat/0608652.pdf

Coppersmith, S. N, Seungwon Lee, and Paul von Allmen. "Exchange in a Silicon-Based Quantum Dot Quantum Computer Architecture." arxiv.org. 25 August 2004. Cornell University Library. 20 October 2013. http://arxiv.org/pdf/cond-mat/0408542.pdf

Cory, D. G. *et al*. "Ensemble Quantum Computing by NMR Spectroscopy." Proceedings of the National Academy of Sciences 94.5 (1997): 1634-639. Web.

Das Sarma, S. and Rogerio de Sousa. "Electron Spin Coherence in Semiconductors: Considerations for a Spin-Based Solid State Quantum Computer Architecture." arxiv.org. 10 February 2003. Cornell University Library. 24 January 2014. http://arxiv.org/pdf/cond-mat/0203101.pdf

Dunn, Michael J. et al. "Quantum Logic As Motivated By Quantum Computing." arxiv.org. 7 December 2004. Cornell University Library. 13 October 2013. http://arxiv.org/pdf/math/0412144.pdf

Dyakonov, M.I. "Revisiting the Hopes for Scalable Quantum Computation." arxiv.org. 5 November 2012. Cornell University Library. 25 October 2014. http://arxiv.org/pdf/1210.1782.pdf

Economou, Sophia E. et al. "A Scalable Qubit Architecture Based on Holes in Quantum Dot Molecules." arxiv.org. 25 September 2012. Cornell University Library. http://arxiv.org/pdf/1209.5469.pdf

Fowler, Austin G. "Time-optimal quantum computation." arxiv.org. 5 February 2013. Cornell University Library. 19 October 2013. http://arxiv.org/pdf/1210.4626.pdf

García-Mata, I., K. M. Frahm, and D. L. Shepelyansky. "Shor's Factorization Algorithm with a Single Control Qubit and Imperfections." arxiv.org. 25 September 2008. Cornell University Library. 27 September 2013. http://arxiv.org/pdf/0809.4416v1.pdf

Green, Alexander S. et al. "Quipper: A Scalable Quantum Programming Language." arxiv.org. 11 April 2013. Cornell University Library. 18 June 2014. http://arxiv.org/pdf/1304.3390.pdf

Guo, H., L. Gui-Lu, and Y. Sun. "Effects of Imperfect Gate Operations in Shor's Prime Factorization Algorithm." arxiv.org. 4 April 2007. Cornell University Library. 27 September 2013. http://arxiv.org/pdf/0704.0516.pdf

Hornibrook, J.M. *et al.* "Cryogenic Control Architecture for Large-Scale Quantum Computing." arxiv.org. 8 September 2014. Cornell University Library. 12 September 2014. http://arxiv.org/pdf/1409.2202v1.pdf

Huang, W., and Zhaohui Wei. "Efficient One-Way Quantum Computations for Quantum Error Correction." arxiv.org. 18 August 2008. Cornell University Library. 30 October 2013. http://arxiv.org/pdf/0712.2295v2.pdf

Jefferson, J.H. et al. "Two-Electron Quantum Dots as Scalable Qubits." arxiv.org. 4 September 2002. Cornell University Library. 16 April 2014. http://arxiv.org/pdf/quant-ph/0206075.pdf

Khodjasteh, Kaveh, and Lorenza Viola. "Dynamically Error-Corrected Gates for Universal Quantum Computation." arxiv.org. 10 June 2013. Cornell University Library. 23 October 2013. http://arxiv.org/pdf/0810.0698.pdf

Kosut, Robert L., Alireza Shabani, and Daniel A. Lidar. "Robust Quantum Error Correction via

Convex Optimization." arxiv.org. 18 September 2007. Cornell University Library. 30 October 2013. http://arxiv.org/pdf/quant-ph/0703274v2.pdf

Kyaw, T.H. et al. "Scalable Quantum Random-Access Memory with Superconducting Circuits." arxiv.org. 23 April 2014. Cornell University Library. 19 August 2014. http://arxiv.org/pdf/1404.5778v1.pdf

Lages, J. and D. Shepelyansky. "Suppression of Quantum Chaos in a Quantum Computer Hardware." arxiv.org. 14 October 2005. Cornell University Library. 16 October 2013. http://arxiv.org/pdf/cond-mat/0510392v1.pdf

Lanyon, B.P., et al. "Experimental Demonstration of Shor's Algorithm with Quantum Entanglement." arxiv.org. 16 December 2007. Cornell University Library. 4 October 2013. http://arxiv.org/pdf/0705.1398.pdf

Lu, C.Y. et al. "Demonstration of Shor's Quantum Factoring Algorithm Using Photonic Qubits." arxiv.org. 16 December 2007. Cornell University Library. 4 October 2013. http://arxiv.org/pdf/0705.1684.pdf

Marshall, Kevin and Christian Weedbrook. "Device-Independent Quantum Cryptography for Continuous Variables." arxiv.org. 27 May 2014. Cornell University Library. 27 October 2014. http://arxiv.org/pdf/1405.6983.pdf

Martín-López, Enrique et al. " Experimental Realisation of Shor's Quantum Factoring Algorithm Using Qubit Recycling." arxiv.org. 24 October 2012. Cornell University Library. 13 October 2013. http://arxiv.org/pdf/1111.4147.pdf

Mor, Tal and Nadav Yoran. "Methods for Scalable Optical Quantum Computation." arxiv.org. 14 March 2006. Cornell University Library. 13 October 2013. http://arxiv.org/pdf/quant-ph/0603118.pdf

O'Brien, Jeremy L. "Optical Quantum Computing." arxiv.org. 11 March 2008. Cornell University Library. 23 October 2013. http://arxiv.org/pdf/0803.1554.pdf

O'Gorman, Joe et al. "A Silicon-Based Surface Code Computer." arxiv.org. 7 October 2014. Cornell University Library. 20 July 2014. http://arxiv.org/pdf/1406.5149.pdf

Omkar, S., R. Srikanth, and Subhashish Banerjee. "Online Characterization of Quantum Dynamics using Quantum Error Correction." arxiv.org. 5 May 2014. Cornell University Library. 18 October 2014. http://arxiv.org/pdf/1405.0964.pdf

Petta, J.R. et al. "Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots." sciencemag.org. 30 September 2005. Science. 19 October 2013. http://pettagroup.princeton.edu/publications/2005/Science_309_2180_2005.pdf

Pla, Jarryd J. et al. "A Single-Atom Electron Spin Qubit in Silicon." arxiv.org. 20 May 2013. Cornell University Library. 5 August 2014. http://arxiv.org/ftp/arxiv/papers/1305/1305.4481.pdf

Pudenz, Kristen L. et al. "Error Corrected Quantum Annealing with Hundreds of Qubits." arxiv.org. 31 July 2013. Cornell University Library. 12 July 2014. http://arxiv.org/pdf/1307.8190.pdf

Qu, Ri et al. "New Approach to Quantum Error Correction." arxiv.org. 24 April 2014. Cornell University Library. 9 September 2014. http://arxiv.org/ftp/arxiv/papers/1401/1401.0456.pdf

Reichardt, Ben W., and Lov K. Grover. "Quantum Error Correction of Systematic Errors Using a Quantum Search Framework." arxiv.org. 28 June 2005. Cornell University Library. 30 October 2013. http://arxiv.org/pdf/quant-ph/0506242.pdf

Sau, Jay D., Sumanta Tewari, and S. Das Sarma. "Universal Quantum Computation in a

Semiconductor Quantum Wire Network." arxiv.org. 24 November 2010. Cornell University Library. 23 October 2013. http://arxiv.org/pdf/1007.4204.pdf

Shankar, Ravi, Swathi S. Hegde, and T.S. Mahesh. "Quantum Simulations of a Particle in One-Dimensional Potentials Using NMR." arxiv.org. 12 July 2013. Cornell University Library. 8 June 2014. http://arxiv.org/pdf/1307.3318v1.pdf

Sornborger, Andrew T. "Quantum Simulation of Tunneling in Small Systems." arxiv.org. 7 February 2012. Cornell University Library. 20 October 2013. http://arxiv.org/pdf/1202.1536.pdf

Uskov, Dmitry B. et al. "Maximal Success Probabilities of Linear-Optical Quantum Gates." arxiv.org. 22 November 2008. Cornell University Library. 23 October 2013. http://arxiv.org/pdf/0808.1926.pdf

Vandersypen, Lieven M.K. et al. "Experimental Realization of Shor's Quantum Factoring Algorithm Using Nuclear Magnetic Resonance." arxiv.org. 30 December 2001. Cornell University Library. 6 July 2014. http://arxiv.org/pdf/quant-ph/0112176v1.pdf

Veldhorst, M. et al. "An Addressable Quantum Dot Qubit with Fault-Tolerant Control Fidelity." arxiv.org. 8 July 2014. Cornell University Library. 8 June 2014. http://arxiv.org/pdf/1407.1950.pdf

Waldherr, G., et al. "Quantum Error Correction in a Solid-State Hybrid Spin Register." arxiv.org. 30 September 2013. Cornell University Library. 27 October 2013. http://arxiv.org/pdf/1309.6424v2.pdf

Wang, Hefeng, Cui-Ping Yang, and Franco Nori. "Robust and Scalable Optical One-Way Quantum Computation." arxiv.org. 27 May 2010. http://arxiv.org/pdf/0912.4016.pdf

Wei, Hai-Rui and Fu-Guo Deng. "Scalable Photonic Quantum Computing Assisted by Quantum-

Dot Spin in Double-Sided Optical Microcavity." arxiv.org. 18 July 2013. Cornell University Library. 15 September 2014. http://arxiv.org/pdf/1306.0053.pdf

Weinstein, Yaakov S. "Quantum Error Correction Implementation After Multiple Gates." arxiv.org. 23 July 2013. Cornell University Library. 27 October 2013. http://arxiv.org/pdf/1305.2763.pdf

Xue, Pang. "Universal Quantum Computing with Semiconductor Double-Dot Molecules on a Chip." arxiv.org. 18 January 2010. Cornell University Library. 18 August 2014. http://arxiv.org/pdf/0908.3719.pdf

Zhang, Jian-Qi et al. "The Scalable Quantum Computations Based on Quantum Dot Systems." arxiv.org. 20 January 2011. Cornell University Library. 24 January 2014. http://arxiv.org/pdf/1101.1818.pdf

Zhao, Bo et al. "A Millisecond Quantum Memory for Scalable Quantum Networks." arxiv.org. 31 July 2008. Cornell University Library. 25 October 2014. http://arxiv.org/pdf/0807.5064.pdf